Claims

1. A method of fragile watermarking a digital image, including the steps of extracting from the image a portion thereof A and generating at least a first ill-conditioned operator related to values extracted from the portion A.

2. A method of fragile watermarking according to claim 1 wherein the ill-conditioned operator is generated by altering a value to increase the operator's condition number.

3. A method of fragile watermarking according to claim 1 or 2, comprising the step of replacing a non-zero singular value of a singular value matrix S_A of an image or portion thereof A, with a solution to a linear equation comprising the ill-conditioned operator.

4. A method of fragile watermarking according to claim 3, wherein the non-zero singular value to be replaced is the smallest non-zero singular value $S_r(A)$ in a singular value matrix S_A of rank r.

5. A method of fragile watermarking according to any one of the preceding claims, wherein a non-zero singular value of a singular value matrix S_W of a watermark pattern or portion thereof W is replaced, such that said replacement increases the condition number of the singular value matrix S_W of the watermark pattern or portion thereof W.

6. A method of fragile watermarking according to claim 5, wherein the non-zero singular value to be replaced is the smallest non-zero singular value $S_t(W)$ in a singular value matrix S_W of rank t.

7. A method of fragile watermarking according to any one of the preceding claims, wherein the step of calculating a replacement non-zero singular value of singular value matrix S_W of a watermark or portion thereof W comprises calculating substantially the following equation part:

$$s_t(W) = \varepsilon,$$

where ε is a small positive real number that increases the condition number of the singular value matrix S_W.

8. A method of fragile watermarking according to any one of the preceding claims, wherein the step of generating at least a first ill-conditioned operator comprises calculating substantially the following equation part:

$$B = \hat{A}\hat{W},$$

where $\hat{W}$ is substantially constructed according to $\hat{W} = U_w\hat{S}_wV_w^T$, $\hat{S}_w$ comprising at least one altered singular value $s_t(W) = \varepsilon$, and such that B forms a parametric family of matrices $B(\hat{s}_r) = \hat{A}(\hat{s}_r)\hat{W}$ for possible values of $\hat{s}_r(A)$.

9. A method of fragile watermarking according to claim 8, wherein $s_r(A)$ is determined by an L_2-norm solution of the least squares problem $\min_{x \in \Re^p} \|Bx - b\|_2^2$ to equal the square of a predefined key N of predetermined value, where b is an arbitrary vector.

10. A method of fragile watermarking according to any one of the preceding claims, wherein the step of calculating the replacement non-zero singular value of singular value matrix A comprises calculating substantially the following equation part:

$$\min_{\hat{s}_r(A)} \left\{ \sum_{i=1}^{q} \left(u_{B_i}^T b / s_i(B(\hat{s}_r)) \right)^2 - N^2 \right\},$$

where u_{B_i} is the I-th column of the matrix formed with the right singular vectors of B.

11. A method of fragile watermarking according to claim 10, wherein $\hat{s}_r(A)$ further satisfies $\hat{s}_r(A) = \overline{s}_r(A) \in [\max(eps, s_r(A) - \delta), s_r(A) + \delta] = [H_0, H_1]$, where δ is a distortion control and *eps* is machine precision, such that the step of calculating the replacement non-zero singular value comprises calculating substantially the following equation part:

$$\min_{\hat{s}_r \in [H_0, H_1]} \left\{ \sum_{i=1}^{q} \left(u_{B_i}^T b / s_i(B(\hat{s}_r)) \right)^2 - N^2 \right\},$$

with all terms as defined herein.

12. A method of fragile watermarking according to any one of claims 9 to 11, wherein vector b is related to at least a first parameter derived from a portion of an image I other than A.

13. A method of fragile watermarking according to claim 12, wherein for a sequential watermarking process comprising the watermarking of portion $A^{(k)}$ after the watermarking of portion $A^{(k-1)}$, k=1,..,L of L portions, then the step of calculating $b^{(k)}$ for portion $A^{(k)}$ comprises calculating substantially the following equation part:

$$b^{(k)} = \begin{cases} A^{(k)} Z^{(k)} & for\ k = 1 \\ A^{(k-1)} Z^{(k)} & else \end{cases},$$

where $Z^{(k)}$ is a pseudo-random binary vector.

14. A method of fragile watermarking according to any one of the preceding claims, wherein the step of calculating the watermarked image or portion thereof $\hat{A}$ comprises calculating substantially the following equation part:

$$\hat{A} = U_A \hat{S}_A V_A^T$$

where $\hat{S}_A$ comprises at least one replaced singular value, U_A and V_A being left and right singular matrices.

15. A method of fragile watermarking according to any one of the preceding claims, wherein a watermark pattern or portion thereof W is generated by a pseudo-random generator seeded by a key K of predetermined value.

16. A method of fragile watermarking according to claim 15, wherein the values of key K and key N are related.

17. A method of fragile watermarking according to either one of claims 15 and 16, wherein the a watermark pattern or portion thereof W is generated by a pseudo-random generator seeded by a key K of predetermined value, combined with either a single or repeated instance of a logo.

18. A method of fragile watermarking according to any one of the preceding claims, comprising the following steps;

i. generating a K-dependent watermark pattern W from Ω, or recalling a pre-existing one;

ii. constructing a parametric family of matrices $B(\hat{s}_r)$;

iii. estimating a unique parameter $\overline{s}_r(A)$, that minimizes the expression

$$\min_{\hat{s}_r} \left\{ \sum_{i=1}^{q} \left(u_{B_i}^T b / s_i(B(\hat{s}_r)) \right)^2 - N^2 \right\}; \text{ and}$$

iv. estimating the watermarked block $\hat{A} = U_A \hat{S}_A V_A^T$ by setting $\hat{S} = diag(s_1(A), \ldots, s_{r-1}(A), \overline{s}_r(A))$.

19. A method of fragile watermarking according to any one of claims 1 to 17, comprising the following steps;

i. generating a K-dependent watermark pattern W from Ω, or recalling a pre-existing one;

ii. constructing a parametric family of matrices $B(\hat{s}_r)$;

iii. estimating a unique parameter $\overline{s}_r(A) \in [\max(eps, s_r(A) - \delta), s_r(A) + \delta] = [H_0, H_1]$, that minimizes the expression:

$$\min_{\hat{s}_r \in [H_0, H_1]} \left\{ \sum_{i=1}^{q} \left(u_{B_i}^T b / s_i(B(\hat{s}_r)) \right)^2 - N^2 \right\}; \text{ and}$$

iv. estimating the watermarked block $\hat{A} = U_A \hat{S}_A V_A^T$ by setting $\hat{S} = diag(s_1(A), \ldots, s_{r-1}(A), \overline{s}_r(A))$.

20. A method of verifying a fragile watermark, characterised by the step of generating at least a first ill-conditioned operator by altering a value to increase its condition number, said ill-conditioned operator being related to values extracted from a received image or portion thereof A^*.

21. A method of verifying a fragile watermark according to claim 20, characterised by the step of calculating a solution to the least squares problem $\min_{x \in \Re^p} \left\| B^* x - b \right\|_2^2$ where $B^* = A^* \hat{W}$.

22. A method of verifying a fragile watermark according to either one of claims 20 and 21, wherein a positive square-

root N^* of the L_2-norm solution of the least squares problem $\min_{x \in \Re^p} \left\| B^* x - b \right\|_2^2$ is compared with key N; and

the received image or portion thereof A^* comprising the fragile watermark is declared authentic if $\left| N^* - N \right| \leq \tau$, where τ is a threshold value.

23. A method of verifying a fragile watermark according to any one of claims 20 to 22, wherein the step of calculating value N^* comprises calculating substantially the following equation part:

$$(N^*)^2 = \sum_{i=1}^{n} \left(u_{n_i}^T b / s_i(B^*) \right)^2 ;$$

N^* is compared with key N; and

the received image or portion thereof A^* comprising the fragile watermark is declared authentic if $\left| N^* - N \right| \leq \tau$, where τ is a threshold value.

24. Apparatus for fragile watermarking of an image in accordance with a method of any one of claims 1 to 19, comprising generating means for generating at least a first ill-conditioned operator, said ill-conditioned operator being related to values extracted from an image or portion thereof A.

25. Apparatus for validating a fragile watermarked image in accordance with a method of any one of claims 20 to 23, and comprising;

generating means for generating at least a first ill-conditioned operator by altering a value to increase its condition number, said ill-conditioned operator being

related to values extracted from a received image or portion thereof $A^{'}$.